Topic Page: Geometry, Non-Euclidean

Definition: **non-Euclidean geometry** from *The Hutchinson Dictionary of Scientific Biography*

Subject: maths and statistics

Study of figures and shapes in three-or-more-dimensional (or curved) space, in which Euclid's postulates may not apply fully or at all. There are now many forms of non-Euclidean geometry, probably the best known being those propounded by Bernhard Riemann; the first proponents of such systems, however, were Karl Gauss, Nikolai Lobachevsky, and János Bolyai.

Summary Article: **non-Euclidean geometry**
from *The Columbia Encyclopedia*

branch of geometry in which the fifth postulate of Euclidean geometry, which allows one and only one line parallel to a given line through a given external point, is replaced by one of two alternative postulates. Allowing two parallels through any external point, the first alternative to Euclid's fifth postulate, leads to the hyperbolic geometry developed by the Russian N. I. Lobachevsky in 1826 and independently by the Hungarian Janos Bolyai in 1832. The second alternative, which allows no parallels through any external point, leads to the elliptic geometry developed by the German Bernhard Riemann in 1854. The results of these two types of non-Euclidean geometry are identical with those of Euclidean geometry in every respect except those propositions involving parallel lines, either explicitly or implicitly (as in the theorem for the sum of the angles of a triangle).

Hyperbolic Geometry

In hyperbolic geometry the two rays extending out in either direction from a point *P* and not meeting a line *L* are considered distinct parallels to *L*; among the results of this geometry is the theorem that the sum of the angles of a triangle is less than 180°. One surprising result is that there is a finite upper limit on the area of a triangle, this maximum corresponding to a triangle all of whose sides are parallel and all of whose angles are zero. Lobachevsky's geometry is called hyperbolic because a line in the hyperbolic plane has two points at infinity, just as a hyperbola has two asymptotes. The analogy used in considering this geometry involves the lines and figures drawn on a saddleshaped surface.

Elliptic Geometry

In elliptic geometry there are no parallels to a given line *L* through an external point *P*, and the sum of the angles of a triangle is greater than 180°. Riemann's geometry is called elliptic because a line in the plane described by this geometry has no point at infinity, where parallels may intersect it, just as an ellipse has no asymptotes. An idea of the geometry on such a plane is obtained by considering the geometry on the surface of a sphere, which is a special case of an ellipsoid. The shortest distance between two points on a sphere is not a straight line but an arc of a great circle (a circle dividing the sphere exactly in half). Since any two great circles always meet (in not one but two points, on opposite sides of the sphere), no parallel lines are possible. The angles of a triangle formed by arcs of three great circles always add up to more than 180°, as can be seen by considering such a triangle on the earth's surface bounded by a portion of the equator and two meridians of longitude connecting its end points to one of the poles (the two angles at the equator are each 90°, so the amount by which the sum of the angles exceeds 180° is determined by the angle at which the meridians meet at the pole).

Non-Euclidean Geometry and Curved Space

What distinguishes the plane of Euclidean geometry from the surface of a sphere or a saddle surface is the curvature of each (see differential geometry); the plane has zero curvature, the surface of a sphere and other surfaces described by Riemann's geometry have positive curvature, and the saddle surface and other surfaces described by Lobachevsky's geometry have negative curvature. Similarly, in three dimensions the spaces corresponding to these three types of geometry also have zero, positive, or negative curvature, respectively.

As to which of these systems is a valid description of our own three-dimensional space (or four-dimensional space-time), the choice can be made only on the basis of measurements made over very large, cosmological distances of a billion light-years or more; the differences between a Euclidean universe of zero curvature and a non-Euclidean universe of very small positive or negative curvature are too small to be detected from ordinary measurements. One interesting feature of a universe described by Riemann's geometry is that it is finite but unbounded; straight lines ultimately form closed curves, so that a ray of light could eventually return to its source.

See cosmology; relativity.

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