Topic Page: Geometry, Riemannian

Definition: **elliptic geometry** from *The Hutchinson Dictionary of Scientific Biography*

Subject: maths and statistics

System of non-Euclidean geometry developed as the initial form of Riemann geometry, and regarding all geometrical operations as carried out in ‘curved’ space, for example, as though on the surface of an ellipsoid or sphere. A ‘straight line’ is thus defined (then) as the shortest curve (geodesic) on the curved surface joining two points.

Summary Article: **Riemannian geometry**
From *The Penguin Dictionary of Mathematics*

A type of non-Euclidean geometry developed by Riemann in 1854. In Euclidean geometry, the distance between two neighbouring points on a plane is given by a relationship of the form

$ds^2 = dx^2 + dy^2$

where rectangular Cartesian coordinates are used. More generally, the relationship can be written as

$ds^2 = A dx^2 + B\,dx\,dy + C\,dy^2$

where *A, B*, and *C* depend on *x* and *y*.Gauss considered this case and showed that it is possible to determine the curvature at a point intrinsically in terms of *A, B*, and *C*. Riemann generalized this approach into the study of any type of metric space in any number of dimensions. What is now called a *Riemannian space* is a space with n coordinates ($x_1, x_2, \ldots, x_n$) in which the distance between neighbouring points is given by a quadratic form,

$$ds^2 = \sum g_{ij}(x)\,dx_i\,dx_j$$

where the $g_{ij}(x)$ are functions of $x_1, x_2, \ldots, x_n$. In the original form of Riemannian geometry, ds^2 was required to be always positive, although this is not the case in applications to general relativity theory. Usually, the coefficients $g_{ij}(x)$ are taken to have a nonvanishing determinant. The $g_{ij}(x)$ are the components of a symmetric covariant tensor field (the *metric tensor*).In Riemannian geometry, the distance between two points can be determined by an integral of *ds. Riemannian curvature* is defined by an expression involving the metric tensor of the Riemannian space and a tensor known as the *Riemann-Christoffel curvature tensor* after Gauss and Elwin Bruno Christoffel (1829-1900).

Riemannian geometry had a profound effect on the way people thought about geometry and on the development of tensor analysis. It was also essential in the formulation of general relativity and in later attempts to develop a unified field theory. The term is sometimes used in a more restricted sense to describe a particular type of non-Euclidean geometry in which the plane is interpreted as a sphere and a line as a great circle on the sphere. In this form of non-Euclidean geometry, Euclid's parallel postulate is replaced by the postulate that no line can be drawn parallel to a given line through a point lying outside the line. Moreover, Euclid's second postulate (that a line can be extended indefinitely in both directions) is not applicable. This non-Euclidean geometry is also called *elliptic geometry*.

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