Topic Page: Game theory

Definition: **game theory** from *Philip's Encyclopedia*

In mathematics, analysis of problems involving conflict. Initially it was based on the assumption that participants in conflict adopt strategies that maximize personal gain and minimize loss. Later, more complex motivations, such as morality, were included. Applications of game theory include business management, sociology, economics and military strategy. The theory was introduced by French mathematician Émile Borel and developed by John von Neumann in 1928.

Image from: **John von Neumann (right) poses with J. Robert...** in *Encyclopedia of New Media: An Essential Reference Guide to Communication and Technology*

Summary Article: **Game Theory**
from *Encyclopedia of Play in Today's Society*

This article presents general historical information on game theory and represents this topic in what is hoped to be language that is easily understandable to lay people without any prior knowledge of this subject. Also, there are explanatory definitions of general principles involved that have been developed in this field and further explorations of game theory attributes. Basic mathematics behind game theory are briefly touched on in a way that does not require too much technical knowledge for understanding how it works. For further clarification, examples are provided with some figures that demonstrate the applicability of game theory to various life circumstances.

Many may hear the words *game theory* and think that the term is referring to theories of game playing. However, this is not necessarily true, as game theory is about particular aspects of interactions between individuals or animals as they strive to achieve the ultimate outcomes from specific events. We often play "what if" games without being actively aware that we are doing so. Many everyday events can stimulate us to play "Chess matches" involving possible scenarios that depend on our actions in some combination with another party, and there definitely are different results on different occasions. Although this article appears in the *Encyclopedia of Play in Today's Society* and the implementation of game theoretical principles is certainly enjoyable and evokes playful qualities in human minds (at least that can be known), game theory is not simply about games that are played. It is about logical mathematical applications to strategic considerations of conflict scenarios involving player entities (both human and other animals), a method of determining a maximum benefit (or payoff) from the conflict for one or more of the players (via payoff matrices), and theoretical as well as literal implications of such episodes.

Game theory allows us to use formal logic to explain what is not typically discernable but is reliably true. Quite often, an individual who attempts to maximize his or her payoff must also simultaneously minimize the other player's payoff. According to David Barash, professor of psychology and zoology at the University of Washington, it might be said that the existence of other people who compete for similar interests that we want necessitates the reason for having game theory. Essentially, in game theory, players must necessarily account for not only their own interests but also others' behavior.

Formal Study of Game Theory

Game theory is actually the academic conceptualization of behavioral phenomena in strategic situations, typically accounted for by mathematical methods of logic. The formal study of game theory has been occurring since the publishing of John von Neumann and Oskar Morgenstern's 1944 book, *Theory of Games and Economic Behavior*. Game theory is generally considered in the domain of applied mathematics, but it is also primarily applicable to economics, biology, computer science, philosophy, and political science. There are numerous important features of game theory. Historically, game theory has been used by nuclear strategists, notably at the think tank RAND Corporation in California, to rationally calculate potential Cold War scenarios that approached perilous and realistic outcomes for the entire world. Additionally, David Barash mentions in *The Survival Game: How Game Theory Explains the Biology of Cooperation and Competition* that biologists and psychologists have been and continue to be able to model and predict the behavior of living things, and in particular more recent extensions to evolutionary strategies. Anatol Rapoport idealistically suggested in *Fights, Games and Debates* that game theory could contribute the most benefit to humanity by facilitating self-improvement and self-knowledge. Therefore, it may be important to become skilled in effectively managing conflict without violence.

Although extensive scholarly literature has been produced on game theory, the general public has not yet fully realized what it actually is. Paul Walker in the *History of Game Theory* lists two recent instances of public notoriety, albeit still possibly not considered in the “mainstream” media, that have occurred in which game theory was prominently displayed. In 1994, the Nobel Prize in economic sciences was awarded to John Nash, John C. Harsanyi, and Reinhard Selten “for their pioneering analysis of equilibria in the theory of non-cooperative games.” In 2005, the Nobel Prize in economic sciences was awarded to Robert J. Aumann and Thomas C. Schelling “for having enhanced our understanding of conflict and cooperation through game-theory analysis.” Game theory oversimplifies reality, in particular where cooperation and competition are involved, and enhances mathematicians' ability to make a tractable analysis.

Game Theory Models

Making simple models to represent complex phenomena facilitates the process of determining answers to our speculations. In 1950, John Nash proved that every finite game, involving any number of players, has at least one (Nash) equilibrium, though there might not be any that involve only pure strategies for all players. When there is more than one equilibrium, and players cannot make binding agreements, they must try to coordinate to arrive at an equilibrium outcome. Many other criteria for equilibrium selection have been studied (e.g., focal points, subgame perfection, stability, and so on). An outcome is *Pareto optimal* (or *efficient)* if no agent can be made better off than that outcome without making another agent worse off.

There are zero-sum games, in which one player maximizes his or her payoff at the expense of the other player's payoff being minimized. Also, there are non-zero-sum games, in which there may be a joint benefit that emerges out of a conflict situation. Alternatively, there may be a joint loss that occurs for players who decided to act in certain ways that are counterproductive to their own interests, for example, in certain formulations of the classic Prisoner's Dilemma. Therefore, there are different types of strategic situations occurring with different interactions between players or agents. Consequently, other game theoretical approaches consider subjective probabilities that attempt to determine non-zero-sum solutions, whereby individuals proceed in a subjective way of estimating probable choices of others. This essentially enables individuals to reduce strategic interactions to a traditional single-agent decision scenario.

In game theory, rational behavior is assumed to enhance the opportunity for the most preferred payoff. Of course, the other player is simultaneously working toward that as well. Game theory does not care how the payoffs are ranked, just how rational the actions are. It must be noted that game theory is not concerned with the rationality of the value (or utility) undertaken, just in the most effective method. Consequently, even a player who intentionally loses a game gets his or her highest payoff because the criterion for his or her utility was determined to be met. The problem occurring with this phenomenon is that rational individuals must account for others' probable choices, which are also contingent on their choices as well.

The Prisoner's Dilemma

It is important to understanding game theory to briefly describe the following classic scenario. In a two-player, one-shot Prisoner's Dilemma, players can either plead guilty or not guilty to a crime (and both are actually guilty by the evidence). When both plead guilty, this results in the most jail time for each player. When both refuse to plead guilty, they receive the least jail time. One player pleading guilty and implicating the other will result in no jail time for that player, while the implicated player (who still pleads not guilty) will receive the greatest possible sentence. Typically, in this case, game theory rationality induces mutual defection, whereby both players (i.e., “potential” prisoners) are “suckered” in to a less desirable outcome because they suspect the other one will defect first. The Prisoner's Dilemma is just that, a dilemma. In this scenario, two players can either both benefit with the highest payoff (both pleading not guilty), both achieve the lowest payoff (both pleading guilty), or one of them benefits while the other does not (one pleads guilty while the other pleads not guilty). However, individuals' concepts of reality are not fixed. In an iterated Prisoner's Dilemma problem, players subjectively weigh a present advantage against possible future losses. Of course, this brings to mind an oft-repeated aphorism, “A bird in the hand is worth two in the bush.”

There are numerous variations to two-player phenomena. The utility (or value) players place on outcomes affects what payoff (or gain) is ultimately the best in their given situation. When another player is considering the likelihood of your decision, which often is in complete opposition to his or her own preference, the resulting conflict provides an opportunity for a payoff, Iohn von Neumann did demonstrate mathematically in *Theory of Games and Economic Behavior* that when two players interests are in complete opposition, there is always a rational course of action.

Other Game Scenarios

In other scenarios, similar interests are desirable. For example, those who are considering becoming short-term investors in the stock market have to decide if the given stock will be likely to appeal to other investors and lead them to buy stock and concomitantly increase the stock value. A player's objective is to maximize his or her long-term expected payoff. Learning takes place at each stage. The theory for determining maximum utility emanates from the Nash Equilibrium theorem that a set of actions has the property of no player being able to profitably deviate from that given course of action, given the actions of other players (i.e., no player has an alternative course of action that increases their payoff).

A point can be made that game theory acknowledges the other player's viewpoint, which is a separate matter from ascribing to their ideological position. Barash also intimates that sometimes being able to predict particular behavior can influence or cause it to actually occur, in what can be called a self-fulfilling prophecy. This has been referred to as the security dilemma in national security analysis. It is interesting that the Romans had a motto of *Si vis pacem para helium* (if you want peace, prepare for war), which accounted for the longevity of their empire, until they forgot to think about the other players in the scenario and were sacked by invaders. Per game theory, conflict is very likely and usually inevitable.

Game theory is not concerned with the ethics or moralizing of who profits the most and who loses. Barash states that different actions result in payoffs that differ. Individuals optimally strive for the highest possible payoff. However, some individuals value their consequences for particular actions differently and have changing payoffs. Moreover, even the most altruistic players derive a personal, or "selfish," benefit. As William Poundstone wrote, "Shortsighted rationality forces players to subvert the common good."

Definitions

We will define a *game* as having the following aspects:

- There are two or more players involved.
- One or more players make decisions from a certain number of options.
- One decision creates different situations that can affect who makes the next decision and the options available.
- The decisions made by each player may or may not be known by the other players.
- There is an ending point, i.e., the game does not continue forever.
- Each combination of decisions determines a payoff to each player.

There are three different ways in which game theory can be applied. The first application is exploring games on a purely theoretical level and examining the problems that directly arise from the development of game theory. The second application of game theory is analysis of strategic interactions with the purpose of predicting or explaining the actions of the people involved. The third application is analysis of the logical consistency of certain arguments.

Attributes of Game Theory

In this section, the basic concepts in game theory will be discussed. Many times in game theory, a payoff matrix is used to represent a game. A payoff matrix shows the possible strategies available to each player and the payoff, amount of money, points, et cetera, that each player receives for his choice, depending on what the other players do. As shown in Figure 1, this information is put into a matrix, in the form *(C,R)*, where C is the payoff to Player 1 when he plays the C strategy and Player 2 plays the *R* strategy, and *R* is the payoff to Player 2 when he plays the *R* strategy and Player 1 plays the C strategy.

$$\text{Player 2}\quad \begin{array}{c} \\ R_1 \\ \\ R_2 \end{array} \overset{\text{Player 1}}{\begin{array}{cc} C_1 & C_2 \\ \left(\begin{array}{c} (C_{11}, R_{11}) \\ \\ (C_{12}, R_{12}) \end{array} \right. & \left. \begin{array}{c} (C_{21}, R_{21}) \\ \\ (C_{22}, R_{22}) \end{array} \right) \end{array}}$$

Figure 1

Definition: Let C_{ij} be the payoff to Player 1 when he uses his i^{th} strategy of his *m* total strategies and

Player 2 uses his j^{th} strategy of his n total strategies. If $\max_{i£m} \min_{j£n} C_{ij} = \min_{j£n} \max_{i£m} C_{ij}$, then (i,j) is a saddle point.

The saddle point is the best that either player can do, given that his opponent is a rational player. In other words, a saddle point is the element of the game matrix that is both a maximum of the minimums of each row and a minimum of the maximums of each column. A game matrix may have no saddle points, one saddle point, or multiple saddle points. When a saddle point exists, it is equal to the value of the game (i.e., the best outcome that both players can guarantee).

Definition: A zero-sum, or strictly competitive, game is a game in which the interests of the parties are strictly contradictory.

In other words, a zero-sum game is a game where it is impossible for the players to benefit from cooperation. Zero-sum games are named for the fact that we can represent the payments so that the sum of the payments to the players is zero, i.e., $C_{ij} + R_{ij} = 0$, where C_{ij} and R_{ij} are the payoffs to Player 1 and Player 2, respectively, when Player 1 plays i and Player 2 plays j.

The following example illustrates the phenomenon of zero-sum games.

Example 1: Consider a two-person zero-sum game in which one player, the chooser, chooses even or odd, and the other player, the guesser, tries to guess what was chosen. The matrix for this game is shown in Figure 2 below.

		Chooser	
		Odd	Even
Guesser	Odd	(+1, -1)	(-1, +1)
	Even	(-1, +1)	(+1, -1)

Figure 2

In Example 1, since there does not exist a payoff for either player that is both a minimum of its row and a maximum of its column, we can see that there are no saddle points.

We note that in a zero-sum game, the payoff to Player 1 is the opposite of the payoff to Player 2.

Definition: A non-zero-sum, or non-strictly competitive, game is a game in which the interests of the parties involved are not strictly contradictory.

Games that are not zero-sum games are classified as non-zero-sum games. In a non-zero-sum game, it is possible for the players to benefit from cooperation.

Example 2: Consider the two-person non-zero-sum game that occurs when two people are trying to decide what to do. Becky and Blake are planning to go out to eat. They talked about it but did not have time to make a decision. In fact, the only thing decided was that Becky wants to go to the Pasta Place and Blake wants to go to the Local Diner. Both Becky and Blake would rather go to the other place together than to go to their choice alone.

		Blake: Pasta	Blake: Diner
Becky	Pasta	(1,2)	(-1,-1)
	Diner	(-1,-1)	(2,1)

Figure 3

This is a non-zero-sum game because both Becky and Blake would benefit from cooperating with each other. Becky would benefit from going to Local Diner with Blake over going to the Pasta Place by herself. Likewise, Blake would benefit from going to the Pasta Place with Becky over going to the Local Diner by himself.

Basic Mathematics Behind Game Theory

The first attempt to create a mathematical theory of strategy for games was made by Emile Borel in 1921. Although the mathematical theory of games did not receive attention until J. von Neumann and O. Morgenstern published their book *Theory of Games and Economic Behavior* in 1944, Neumann proved the fundamental theorem of game theory, the minimax theorem, in 1928.

The minimax theorem, which is used to analyze zero-sum games, says that an equilibrium pair exists when there is a strategy that is the best for Player 1, regardless of what Player 2 plays, and there is a strategy that is the best for Player 2, regardless of what Player 1 plays. The pair of these two strategies is called an equilibrium pair. When an equilibrium pair exists, it is the value, or solution, of the game.

To create a mixed, or randomized, strategy for Player 1, we need to determine what percentages of the time he should play each of his strategies. A mixed, or randomized, strategy is a probability distribution over the whole set of strategies of a player.

Example 3: Consider two people, Brandon and Tiffany. They both enjoy each other's company, but neither can communicate with the other before deciding whether to stay at home (where they would not see each other) or go to the beach this afternoon (where they *could* see each other). Each prefers going to the beach to being at home and prefers being with the other person rather than being apart. This game can be represented by the following matrix form:

		Tiffany: Home	Tiffany: Beach
Brandon	Home	(0,0)	(0,1)
	Beach	(1,0)	(2,2)

Figure 4

Each player has a set of strategies ([Home, Beach] for both players in this example). Specifying one strategy for the row player (Brandon) and one strategy for the column player (Tiffany) yields an outcome, which is represented as a pair of payoffs.

In this example, going to the beach is a (strictly) dominant strategy for each player, because it always yields the best outcome, no matter what the other player does. Thus, if the players are both maximizing their individual expected utilities, each will go to the beach. So (Beach, Beach) is a dominant strategy equilibrium for this game. Because of this, Tiffany and Brandon, if they are rational, do not need to cooperate (make an agreement) ahead of time. Each can just pursue their own interest, and the best outcome will occur for both.

Example 4: Now consider Kim and Derek. Derek likes Kim, but Kim does not like Derek that much. Each knows this, and neither wants to call the other before deciding what to do this afternoon: stay at their respective homes or go to the neighborhood swimming pool. Here is the matrix form:

		Derek: Home	Derek: Pool
Kim	Home	(2,0)	(2,1)
	Pool	(3,0)	(1,2)

Figure 5

In this case, Kim's best strategy depends on what Derek does. But if she assumes Derek is rational, she will reason that he will not stay home, because going to the pool is a dominant strategy for him. Knowing this, she can decide to stay home. This is called iterated dominance. In this example, Kim gets higher utility than Derek because of their relative preferences, and Derek gets less utility than he would have if Kim wanted to be with him. In this example, Pool-Home (3,0), Home-Pool (2,1), and Pool-Pool (1,2) are all Pareto optimal outcomes. The equilibrium outcomes in both this example and the previous one are Pareto optimal.

Example 5: This is the classic Prisoner's Dilemma problem. Consider Baxter and Jerry, two prisoners who have each been offered a deal to turn state's witness (defect) against the other. They cannot communicate. They had originally agreed to remain in solidarity, i.e., not testify against each other, but since the agreement cannot be enforced, each must choose whether to honor it. If both remain in solidarity, then they will each only be convicted of a minor charge. If only one defects, then the state will throw the book at the other and let the defector go. If they both defect, each will get convicted of a serious charge.

		Jerry: Solidarity	Jerry: Defection
Baxter	Solidarity	(3,3)	(1,4)
	Defection	(4,1)	(1,1)

Figure 6

In this game, the strategy of defection is weakly dominant for each player, meaning that whatever the other player does, defecting yields an outcome at least as good and possibly better than remaining in solidarity would. Note that if the bottom-right cell payoffs were (2,2) instead of (1,1), then defecting would be strictly dominant for each player. Either way, Defection-Defection is a dominant strategy equilibrium. However, it is not Pareto optimal. Both players could be made better off if neither defected against the other.

This is an example of a social dilemma: a situation in which each agent's autonomous maximization of self-utility leads to an inefficient outcome. Such a situation can occur for any number of people, not just two. An agreement by two people to trade with each other (involving goods, services, and/or money) sets up a Prisoner's Dilemma-type game whenever the agreement cannot be enforced.

Example 6: The example of Brandon and Tiffany is now revisited. They are going to the same conference, and each is expecting the other to be there, but they have not seen each other yet. Each has a choice of two activities for the first afternoon: swimming or hiking. They both hope to see each other (if they do not they will have no fun), and each prefers swimming over hiking. They must each decide what to do before knowing where the other is going.

		Tiffany	
		Swim	Hike
Brandon	Swim	(2,2)	(0,0)
	Hike	(0,0)	(1,1)

Figure 7

The best outcome is obviously Swim-Swim, but going swimming is not dominant for either player. Both Swim-Swim and Hike-Hike have the property that each player's strategy is the best (or tied for the best) response to the other player's strategy in that pairing. This defines a more general equilibrium notion called the Nash equilibrium. The dominance equilibria of examples 4-6 are all Nash equilibria as well.

A third equilibrium exists in this game involving what are called mixed strategies. A mixed strategy is a probability distribution over the pure strategies (which are Swim and Hike for each player in this example). Note that the players do not have to have the same set of strategies available to them, even though that has been the case in all the presented examples.

As has been shown, game theory is actually the academic conceptualization of behavioral phenomena in strategic situations, typically accounted for by mathematical methods of logic. There are numerous important features of game theory that have been discussed. This certainly has not be an exhaustive discussion of this topic, nor even close to the technical precision with which stalwart game theorists apply to various scenarios under study at any given point in time. Hopefully, however, this has been an enlightening introduction that informs even the passive reader of at least some of what game theory really involves.

See Also

Human Relationships in Play, Play as Learning, Psychology of, Psychoanalytic Theory and Play, Psychology of Play (Vygotsky)

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Vaughn, Brandon
Roberts, Frank W.
(University of Texas, Austin)

APA

Chicago

Harvard

MLA

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