Definition: Abel, Niels Henrik from *The Columbia Encyclopedia*

(nēls hēn'rik ä'bÔl), 1802–29, Norwegian mathematician. While a student at the Univ. of Christiania (Oslo) he did fundamental work on the integration of functional expressions and proved the impossibility of representing a solution of a general equation of fifth degree or higher by a radical expression. He investigated generalizations of the binomial theorem, pioneered in the general theory of elliptic functions, and showed that elliptic functions are a generalization of trigonometric functions. Commutative groups are also called Abelian groups in his honor. He died of tuberculosis at the age of 26, leaving contributions that rank him as one of the greatest mathematicians of the 19th cent.


Summary Article: VI.33 Niels Henrik Abel

from *The Princeton Companion to Mathematics*

Abel's life was short and penurious, but successful, and he received recognition in his lifetime. His father—a minister of the church in Norway, but also at one time a government minister—overreached himself and when he died he left the family in straitened circumstances. Abel's exceptional intellectual talents were recognized at school, and funds were raised to enable him to complete his education and, in particular, to study mathematics. At age twenty-two, he was awarded a scholarship to make a two-year tour of Europe, during which he studied in Berlin and Paris. In Berlin he met and was befriended by Auguste Crelle, the engineer who had just founded the *Journal für die reine und angewandte Mathematik* (otherwise known as Crelle's Journal). Almost all of Abel's mathematical work was published in the first four volumes of the journal. From 1826 until his death in 1829 Abel eked out a poor existence, earning a little by teaching, but using what few resources he had to support his mother and his younger brother. He died of consumption at the age of twenty-seven within a couple of days of the news reaching Norway that he had been appointed to an established post in Berlin.

Abel's main mathematical contributions lie in three distinct areas. The first of these was the theory of equations. Here he was influenced by ideas published by Lagrange [VI.22] in 1770 and Cauchy [VI.29] in 1815 about the form of functions of the roots of an equation, and what happens to such functions when the roots are permuted. Lagrange had hinted at the possibility that quintic equations might perhaps not be soluble in classical terms and Paolo Ruffini had expended much effort between 1799 and 1814 trying to prove this, though he had not managed to persuade his contemporaries. Abel's first success was to give an acceptable proof of the fact that, for polynomial equations of degree 5, there is no formula in the coefficients involving the usual operations of arithmetic together with extraction of roots that will always yield a solution. This first appeared in 1824 in a short pamphlet written in French and published privately in Christiania (Oslo). Once Abel reached Berlin, however, Crelle translated it into German and published it in the first volume of his journal; he also published a fuller, more detailed account, covering polynomials of any degree greater than 4, in 1826.

Abel returned to equations a few years later, publishing a long paper in 1829 about a class of equations satisfying two special conditions. His first requirement is that every root of the equation can be expressed as a function of every other root, the second that these functions commute (in modern terms, the Galois group [V.21] of the equation is commutative). He proved various theorems about such
equations, the most striking being that they are soluble by radicals. This represented an extensive
generalization of the ideas described by Gauss [VI.26] in the seventh part of Disquisitiones
Arithmeticae, where the special case of cyclotomic equations (which satisfy both of these conditions)
is treated systematically. It was in honour of this work that, later, the adjective "Abelian" was applied to
groups that are commutative. It is important to appreciate, however, that Abel reached his results in
the theory of equations without any appeal to groups, which at that time were not yet known.

He also made major contributions to the theory of convergence. Although there had been over a
century of critical thinking devoted to foundations of the calculus, modern ideas of rigour were only just
emerging in the writings of Bolzano [VI.28], Cauchy, and others. Convergence had received some
attention in Cauchy's lectures of 1820–21, but series in general, and power series in particular, were still
far from well understood. Among other contributions, Abel offered a proper proof of the binomial
theorem for exponents other than positive integers, and the insight about the continuity of a function
defined by a power series as its argument goes to the circle of convergence that is now known as
Abel's limit theorem.

Perhaps his greatest discoveries, however, were in the area where analysis and algebraic geometry
come together. To summarize his legacy in this area in just a few words: first, a new and productive
approach to the theory of elliptic functions [V.31]; and, second, a vast generalization of elliptic functions
to what are now called Abelian functions and Abelian integrals. In this area Abel competed for priority
with Jacobi [VI.35]. Most (though by no means all) of his work was written in two memoirs. One was
published in two parts, "Recherches sur les fonctions elliptiques" and "Précis d'une théorie des
fonctions elliptiques," coming to well over two hundred pages in Crelle's Journal in 1828 and 1829. The
other, entitled "Mémoire sur une propriété générale d'une classe très étendue de fonctions
transcendantes," was submitted to the Paris Academy of Sciences in October 1826. There it lay on
Cauchy's desk, unread until after Abel's death. It was published by the Paris Academy in 1841. The
manuscript itself, however, was stolen by G. Libri, lost, and rediscovered in parts between 1952 and
2000 by Viggo Brun and Andrea del Centina.

In June 1830 the Paris Academy awarded its Grand Prix de Mathématiques jointly to Abel
(posthumously) and Jacobi for their work on elliptic functions.

Further Reading
- Del Centina, A. 2006. Abel's surviving manuscripts including one recently found in London. Historia
  Mathematica 33:224-33.
- Holmboe, B., ed. 1839. Oeuvres Complètes de Niels Henrik Abel, two volumes. (Second edn.: 1881,
  edited by Sylow, L. and Lie, S. Christiania: Grøndahl & Søn).
- Ore, O. 1957. Niels Henrik Abel: Mathematician Extraordinary. Minneapolis, , MN: University of
  translation: 2000, Niels Henrik Abel and His Times: Called Too Soon by Flames Afar, translated by R. H.
  Daly. New York: Springer.).

